An Exploration of Origami Integrated Teaching with Virtual and Physical Manipulatives

Hsi-Hsun Yang¹, Yuan-Ting Chen², Sheng-Kai Yin³,*
¹ National Yunlin University of Science and Technology, Taiwan. jimmy@yuntech.edu.tw
² National Yunlin University of Science and Technology, Taiwan. imdmao@gmail.com
³ Cheng Shiu University, Taiwan. ysk651224@gmail.com

ABSTRACT

Physical origami helps students understand the properties of graphic symmetry, rotation, and folding, improving mathematical learning, geometric reasoning, and conjecturing through mental rotation, spatial orientation and visualization. Using the digital origami simulation system, technology is integrated into learning origami, facilitating physical origami challenges related to precision, paper thickness, and crease elimination. This study constructs the Web of Origami Simulator II (WOOS II) in order to train learner spatial ability, and designs a set of integrated teaching materials with physical origami manipulatives. The technique of origami of one straight cut is used as the basis for teaching material, emphasizing the steps of folding paper, making one straight cut, and observing the shapes when unfolded.

This experiment adopts the one-group pretest-posttest design, with teaching materials integrated into virtual and digital manipulatives applied to 41 fifth-grade students in an elementary school in central Taiwan. Resulting experimental data are collected, analyzed, and discussed. Results show better performance in WOOS II and physical origami relates to better manifestation of learner spatial ability. Analysis of the learning sheet shows the line-symmetric and spatial abilities influenced the students’ performances in origami of one straight cut. The digital origami simulation environment can help students grasp the concept and operation of origami of one straight cut, while deepening learning through the parallel application of physical origami. Learning of spatial, geometric, and line-symmetric concepts is also improved.

Keywords: Origami, Computer Assisted Learning System, Spatial Ability, Virtual Manipulatives, Physical Manipulatives, Symmetry

1. Introduction

Moyer (1978) pointed out that inadequate spatial ability hinders pupils from understanding a numerical system. According to Homan (1970), perceptual skills were one of the factors influencing students’ mathematical learning. Lack of perceptual skills, such as spatial relationship, distance, and size, relationship presented obstacles to students in activities related to measurement, estimation, problem solving, and geometry. Spatial ability can affect the development of mathematical learning (Kali & Orion, 1996). Likewise, and Tso and Liang (2001) indicated spatial ability can affect geometric learning.

In addition to traditional approaches to teaching geometry, some researchers have employed origami to teach geometric skills (Cipoletti & Wilson, 2004; Chen, 2005; Hull, 2002). Tam and Chen (2009) tried using origami to overcome learner fear of ruler-and-compass construction. The National Council of Teachers of Mathematics encourages students to learn mathematical concepts through hands-on experiences (NCTM, 2000; Boakes, 2009; Cakmak, 2009). Origami is one such teaching activity. Origami trains coordination skills, such as hand-eye-brain coordination (Chan, 2007). Skills applied during the process of origami include geometric and line-symmetric concepts, as well as mental rotation. In Taiwan, origami is used in elementary mathematical courses to teach students the properties of line-symmetry and the angle bisector theorem—basic concepts for junior and senior high school geometric textbooks (Ministry of Education, 2008). The concept of symmetry derives from geometric transformation, including relative graphic positions in space, as well as congruent and symmetrical relationships (Libeskind, 2008). In Taiwan elementary schools, symmetry instruction includes graphic cutting, drawing, and characteristics of perpendicular bisector symmetric point of symmetry axis. Such knowledge is fundamental, yet some students have weak spatial ability, unable to mentally process visual graphs or understand graph symmetry. As pointed out by Tso (2003) and Liu and Liu (1994), when learning line-symmetric graphs, students were only familiar with the top/bottom and left/right symmetric graphs,
finding it difficult to identify the symmetry axis of other symmetric graphs. All such knowledge can be grasped through origami.

Physical origami teaching is an intuitive method to help students in learning through visual observations and operations, along with transform of abstract geometric graphs into tangible origami (Arici & Tutak, 2015; Pope, 2002). However, during physical origami teaching, the multiple rotations and complex steps can easily confuse students. When implementing physical origami, students with inadequate finger dexterity have difficulty folding paper precisely. This can lead to failure to finish the origami (Tam and Chen, 2009).

Origami teaching can be extended into a digital environment, easing teacher burden. This approach decreases costs of learning tools (Izydorczak, 2003; Suh, Moyer & Heo, 2005). Using computer images to simulate physical manipulatives is referred to as virtual manipulation, which can provide an operating interface for teachers and students via a mouse. When virtual manipulation is integrated into teaching, it presents abstract concepts in a visually dynamic manner. Virtual manipulatives represent abstract concepts, bridging the gap between tangible objects, graphs, and symbols. This approach facilitates learner understanding (Moyer, Bolyard, & Spikell, 2002; Reimer & Moyer, 2005). In addition, the dynamic graphs within a digital environment can easily represent 3-dimensional changes. This feature helps students comprehend the origami steps and processes, while allowing adjustments as necessary (Lu & Lin, 2013). Origami digitalization improves origami precision (Yang, Yin, & Chen, 2014) and avoids errors, such as those arising from the multiple folds, common in physical origami. This approach also avoids the limitation of paper thickness, allowing the paper to be folded multiple times, and with more variation in shapes. Within a digital environment, students can practice repeatedly without using paper, reducing environmental waste (Yang & Chen, 2016). A digital environment, however, cannot present a stereoscopic origami process. The training of finger and muscle dexterity and coordination is also missing from a digital exercise.

The effects of using virtual manipulatives are directly influenced by several factors, including manipulative design, teacher experience, teaching units, and course design. Some literature reports virtual manipulatives have a better effect than traditional physical manipulation (Suh, Moyer, & Heo, 2005; Moyer, Niezgoda, & Stanley, 2005; Steen, Brooks, & Lyon, 2006; Moyer, Salkind, & Bolyard, 2008). However, contrary results have also been reported, finding no significant difference between virtual and traditional physical manipulatives (Drickey, 2000). A few researchers have pointed out that simultaneous use of both physical and digital manipulatives had better effects than the single use of either (Terry, 1995; Ball, 1988). Therefore, this domain deserves to be further explored and understood. The current study proposes adopting integrated origami teaching with virtual and physical manipulatives, probing the influence of such teaching on students and laying the foundation for future improvement.

Given the abovementioned reasons, this study design includes a set of virtual and physical manipulatives focused on the technique of origami of one straight cut—a the technique of folding a sheet of paper, cutting the paper along a straight line, and observing the geometric shape that results after the paper is unfolded (Figure 1). Limited to one straight cut, students are required to explore how to create various geometric shapes with the single cut. This training involves repeated application of line-symmetric concepts, not limited to the vertical, horizontal, or tilted symmetry axis, while requiring attention to each origami step’s influence on the final shape. During the process, students learn about line-symmetric and geometric relationships of two-dimensional shapes, while applying mental rotation and spatial orientation in order to extrapolate resulting shapes after folding and cutting. All these concepts are important in mastering spatial and geometric proofs. Though origami has been used to train students’ spatial ability (Boakes, 2009; Tam and Chen, 2009), there are no related teaching materials targeted at training concepts like line-symmetry and spatial relationships.

![Figure 1. Origami of One Straight Cut](image)

The technique of origami of one straight cut, as adopted in this study, is limited to making one cut along a straight line. It is more difficult for a digital system to simulate curved cutting. Elementary school students are also challenged by curved cuts—possibly above their cognitive development stage. Therefore, this study focuses exclusively on single straight cuts.

This study employs the Web of Origami Simulator II (WOOS II), an incremental improvement on WOOS I (Yang, Yin, & Chen, 2014). The WOOS II (virtual manipulative) system, as with physical origami of one straight
Spatial ability is an important skill for both school settings and general life. When students solve geometric problems, their spatial visualization and spatial orientation (the two constructs of spatial ability) are positively correlated with the effects of their learning about geometric problem solving (Tso & Liang, 2001). Ambrose and Falkner (2002) used shapes familiar to students, such as the triangle, quadrangle, and pentagon, as teaching materials to develop spatial concepts of elementary school students in lower grades. Likewise, Nilges and Usnick (2000), and Boakes (2009) indicated that spatial ability would influence the students’ acquisition of spatial ability, geometry, measurement, and numerical concepts.

Since spatial ability can play such a key role, it is important to first clearly define the term? Kelley (1928) considered special ability to be a kind of visual cognition and memory. Thurstone (1938) and Kelley (1928) shared an identical view of this. As defined by Thurstone (1938), spatial ability references the ability to mentally remember an image, shift and rotate the image, and generally create and apply images in the mind. Spatial ability is classified in different ways across scholars. Based on the spatial ability classification of McGee (1979), Linn, and Petersen (1985), and Lohman (1988), the current study classifies spatial ability into three categories: spatial orientation, spatial visualization, and mental rotation. Spatial orientation refers to quick mentally identification of the spatial elements of objects, such as direction, position, and angle. Spatial visualization refers to mentally creation or identification of object shapes after two-dimensional or three-dimensional objects are folded or unfolded. Mental rotation refers to quick and precise mental rotation of spatial imagines.

The origami process includes 2D to 3D conversion, which entails the application of spatial ability. From this study’s perspective, students must apply the spatial orientation skill, of spatial ability, to observe the symmetry axis of the graph. This includes considering how to fold the paper. Spatial visualization is employed to identify the shape of the paper, while mental rotation is used in order to imagine the post-cutting shape. Cakmak, Isiksal, and Koc (2014) also have shown that origami instruction positively impacts the spatial ability of elementary school students. Chen (2012) used origami to inspire students with creativity and observed students’ development of graphic creativity, finding that students could see and create different shapes during the origami process. Li (2014) applied origami teaching to performance assessment, integrating mathematical problems into the process of making diagonal folds, in order to teach multiplication formula and the Pythagorean theorem. Yang and Yin (2015) utilized origami to develop the concept of symmetry axis and improve student ability with geometric conjectures.

When origami is used as a teaching tool, different creases and geometric shapes are created in the folding process. For example, when students learn folding line-symmetric graphs, they should be able to judge whether the shapes on both sides of the symmetry axis are equal and be able to mentally remember and maintain these shapes. All such activities involve the application of spatial ability.

2.2 Origami’s Fold and Cut

When discussing the fold-and-cut problem (Shen, 1721), Demaine, Demaine, & Lubiw (1998) found that the symmetry axis of the graph was key to solving the problem. The straight-skeleton method produced a correct folding line, and just one straight cut could create the shape. For complex shapes, the folding process is very complicated. In another research that created different shapes with the symmetry axis, Wang and Tzeng (2015) discussed paper folding and cutting in terms of two-dimensional shapes. According to their research, the post-cutting shape arrangement correlates with the position of the folding line. Two basic graphs, the triangle and square, are categorized for paper folding. When the paper is folded in the same way as these two basic graphs, regular shapes can be cut out.

Taiwan’s elementary and junior school mathematical courses emphasize teachers helping students learn line-symmetric graphs through paper folding and cutting activities (Ministry of Education, 2008). Related studies show tilted symmetry axis is more difficult than either vertical or horizontal symmetry axis. Elementary pupils tend to be clear about the property of left-right symmetry, second to which is top-bottom symmetry. However, such students are relatively unfamiliar with tilted symmetry (Liu, Sheu, Yih, Juan, & Liu, 1994). In the current study, the technique of origami of one straight cut
specifically refers to folding the paper several times, making a single straight-line cut, and unfolding the paper, resulting in a geometric shape. With such teaching material, students learn to predict the post-cutting shapes; each way of folding and cutting generates different geometric shapes. The angle and step sequence of paper cutting influences the final geometric shape (Wang & Tzeng, 2015; Yang, Chen, & Yin, 2016). For such operations, students must apply their spatial visualization, spatial orientation, and mental rotation skills, as well as their understanding of line-symmetry, to judge the correct angle of folding. These concepts are a crucial foundation to be applied in junior and senior high school courses, as well as daily life.

2.3 Physical Manipulative and Virtual Manipulative

Easy to operate, physical manipulatives have long been an important tool to assist teachers with teaching. For mathematical teaching, physical manipulatives are tangible objects that connect previous concepts. This helps students construct, enhance, and link numerous mathematical representations (Taylor, 2001; Drickey, 2000; Clements, 1999). Physical manipulatives can be touched, rotated, repositioned, and collected (Perl, 1990). The common physical manipulatives include the tangram, dice, building blocks, figure cards, alphabet cards, charts, set squares, protractors, and compasses (Suh, Moyer & Heo, 2005). Most researchers recognize the positive effects and functions of physical manipulatives in teaching mathematics through operating and utilizing other representations (Moyer & Westenskow, 2013; Kim, 1993). For instance, Tam and Chen (2009) helped students solve problems related to ruler-and-compass construction via the practice of physical origami, with the six basic origami movements improving through Huzita-Hatori axioms. The creases in the paper folding process and the folded overlapped line segments or angles serve as the tools to solve ruler-and-compass construction problems. Chen (2012) inspired students with cutting influence and helped students simulate mental rotation. However, some empirical analysis has exposed problems, such as the lack of physical manipulatives, the difficulty in storing physical manipulatives (due to large volume), the difficulty in class management, the limitations of size and quantity, the high costs of materials, and less varied resources (Chou & Lin, 2010). Moyer (2001) discovered that operationally physical manipulatives did not have sufficiently clear connections with abstract mathematical symbols.

In recent years, rapidly changing computer software and hardware technology, as well as the popularization of networks, has brought numerically intense computations into easy availability. Manipulatives do not only involve the implementation of physical objects any more as they previously did. The National Council of Teachers of Mathematics (2000) emphasized scientific and technological products are indispensable to modern mathematical instruction. Therefore, many manipulatives can presented through information technology equipment and multimedia channels (Moyer, Bolyard, & Spikell, 2002). As pointed out by Kim (1993), virtual manipulatives simulate physical manipulatives and replace the manual operation of objects via a keyboard operation. Such cases retain properties of physical manipulatives are referred to as virtual manipulatives. Currently, a large amount of evidence supports the positive effects of virtual manipulatives (Li & Ma, 2010; Moyer & Westenskow, 2013). Char (1989) stated that one single type of manipulative cannot be applicable to all students and instructional situations. Different virtual manipulatives satisfy the instructional needs of different students. Izydorczak (2003) summarized eight main advantages of virtual manipulatives, including that virtual manipulatives are easier to operate and more extendable than physical manipulatives. Virtual manipulatives provide guidance, give real-time feedback, monitor learning activities automatically, and reduce teacher homework correction workload (Durmus & Karakirik, 2006; Suh, Moyer & Heo, 2005).

Current existing origami-related virtual manipulatives include the Origami Club (2002) (an origami website with animated instruction) the Open Media Lab of Chukyo University produced by Miyazaki, Yasua, Yokoi, and Toriwaki (1996) (a set of origami simulators) the Origami Simulator of Takumi (2015) (designed by David in 2008) as well as the Make-a Flake (2016) (an educational game based on paper folding and cutting). While all such virtual manipulatives can promote understanding of origami, they are not sufficient to serve the purpose of true manipulatives.

Numerous researchers have shown both virtual and physical manipulatives have their own respective advantages (Suh, Moyer & Heo, 2005, 2005; Moyer, Niezgoda, & Stanley, 2005; Steen, Brooks, & Lyon, 2006; Moyer, Salkind, & Bolyard, 2008; Drickey, 2000). Some researchers have pointed out it is better to use both types of manipulatives simultaneously than to use either
of them individually (Terry, 1995; Ball, 1988). In addition to interactive features of virtual manipulatives, developers must be able to emphasize the learning goals (Leathrum, 2001), staying within a cognitively appropriate learning context. Therefore, this study proposes that origami instruction should integrate both virtual and physical manipulatives, adopting different teaching methods at different stages, in the belief that both physical operation and virtual simulation help learners.

3. Research Method and Experimental Design

3.1 Experimental Design and Process

This study adopts the quasi-experimental one-group pretest-posttest design. Subjects include 41 fifth-grade students from two classes in an elementary school. The experiment is conducted using the technique of origami of one straight cut. The first session includes an origami of one straight cut written pre-test. The second to fourth sessions (40 minutes per session) include teaching activities of digital origami employing the one straight cut technique, where the schoolchildren operate the WOOS II software. The fifth session is a teaching activity including the learning sheet of physical origami of one straight cut. The sixth session focuses on self-examination and discussion. The final session includes origami of one straight cut written post-test. Test scores are then collected, statistically analyzed, and discussed.

3.2 Materials

3.2.1 Teaching materials

Wang and Tzeng (2015) studied the patterns of origami creation, finding that multiple geometric shapes can be cut with triangular and square origami. Thus, this study extends these two geometric shapes and classifies the folded paper into the two categories of triangle and rectangle, as shown in Figure 2. After further folding, one straight cut produces the basic L shape or the basic V shape. Other shapes created by one straight cut are derived from these two basic shapes. For example, the shapes on the left side of Figure 2, the shapes of T, □ and 王, are developed using the symmetry of the L shape. Regarding the shapes on the right side of Figure 2, the W shape is generated by copying the left-right symmetry of the basic V shape; the same rule applies to the ★ shape. When performing origami of one straight cut, students must be familiar with the relation of two-dimensional graphs and line-symmetry.

Figure 2. Basic Shape of Origami of One Straight Cut

Figure 3 shows the origami of one straight cut process to form the □ shape. As shown in Figure 3, the □ shape is comprised of 4 line-symmetric L shapes. The paper is first folded along the symmetry axes of these four L shapes, respectively, and then cut in the L shape, producing the □ shape (Figure 4). Therefore, the origami of the one straight cut technique can help students learn line-symmetry, spatial ability, and graphic conjecture.

Figure 3. Deconstruction figure of the □ Shape

Additionally, origami of one straight cut includes more than one way of folding for each resulting shape. For instance, there are two ways of folding the above-mentioned □ shape. First, the □ shape can be considered as the mirror of 4 small L shapes. Thus, the paper is folded in half twice, into the rectangular shape, and then, folded in the L shape. Second, the □ shape can be considered as the mirror of 2 big L shapes. Thus, the paper is folded in half once into the triangular shape, and then, folded into the L shape. The other shapes are extended from the □ shape, such as the □.
3.2.2 The digital origami simulation system

The WOOS II program was derived from WOOS I (Yang, Yin, & Chen, 2014). WOOS II allows manipulation of multiple origami directions, as well as the new functions of paper cutting, ruler, and protractor. The WOOS II program combines the functions of origami animation with clicking operations. WOOS II’s origami of one straight cut teaching materials are designed in accordance with the competence indicators of Grades 1-9 Curriculum Guidelines (Ministry of Education, 2008). WOOS II consists of 8 missions, the contents of which are designed with increasing difficulty. The 8th mission is a general review, and the students must finish the current mission before moving onto the next one. Each level includes a test consisting of operation questions, gap questions, and multiple-choice questions. A perfect score for each level is 100.

In addition, steps shown in Figure 4 show the WOOS II interface, which represents a 3D image of origami users can manipulate. Figure 5 shows the four areas interface areas:

A (Question Area): This area mainly holds the questions. The students must read the questions and understand the main points.

B (Gizmos Area): The area provides various tools, such as a ruler, protractor, and scissors to assist students in learning. There is also a button to return to homepage.

C (Operating Area): The virtual paper to be manipulated is displayed here. Students manipulate the paper by clicking and dragging with the computer mouse.

D (Answer Area): Answers to questions are submitted in this space.

Teaching activities are carried out across 3 sessions. The first session executes missions 1-4 in order to test learner familiarity with line-symmetric graphs. Specifically, students are required to identify a line-symmetric graph and practice the concept of corresponding angles. The second session introduces missions 5-6, teaching geometric properties of the triangle and quadrangle, as well as the basic shapes of origami of one straight cut. These materials will enable a learner to deconstruct geometric graphs into basic shapes through mental rotation and the line-symmetric concept. The third session introduces missions 7-8 that practice different geometric shapes, building upon concepts learned in previous missions. Manipulation and animation are provided so students can practice creating results from the one straight cut process.

3.2.3 Teaching materials for physical origami of one straight cut

After the WOOS II learning activities, this study applies physical origami of one straight cut activities. Eight questions are selected and revised in the learning sheet for physical origami of one straight cut (as shown in Figure 6).

First, the teacher demonstrates the operation of origami of one straight cut. The English letter X is taken as an example. Step 1: graph X is drawn on physical colored paper (Figure 7). Step 2: the line-symmetric concept is applied to determine the symmetry axes of graph X. As easily seen, there are 4 symmetry axes of X. The paper is first folded along any one symmetry axis. Next, the learner searches for other symmetry axes. Step 3: all the lines drawn in Step 1 are overlapped, and one single cut is made along the line. After the above-mentioned demonstration, students begin to respond to the learning sheet questions. As directed by the questions, students next paste the finished colored paper onto the learning sheet. Full marks are 100 for a complete learning sheet of physical origami of one straight cut. How well the students learned the one straight cut is judged according to the creases and shapes created by the
students. In the case of students not cutting the paper with one straight cut, points are deducted.

![Image](image_url)

**Figure 7. Demonstration of Physical Origami of One Straight Cut Operation-The English Letter X**

3.2.4 Origami of one straight cut test

In order to observe the impact of digital origami instruction on students’ spatial ability, this study adopts variables from theory of spatial ability, and categorized spatial ability into spatial orientation, spatial visualization and mental rotation. These indicators are next aligned with competence indicators of Grade 1-9 from the Curriculum Guidelines of the Ministry of Education (2008), as shown in Table 1. Cronbach's α of the pretest is 0.803 and Cronbach's α of posttest is 0.822, demonstrating the assessment tool’s consistency and reliability. Origami of one straight cut test questions include four items, each made up of five questions. The full score for each question is 5 points, totaling 100. The test time is 40 minutes.

Table 1. Origami of one straight cut test, corresponding spatial ability and competence indicators of Grade 1-9 Curriculum Guidelines

<table>
<thead>
<tr>
<th>Item</th>
<th>Corresponding competence indicators</th>
<th>Corresponding spatial ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation of meaning of graphic symmetry</td>
<td>S-2-06, S-3-03</td>
<td>Spatial orientation, mental rotation</td>
</tr>
<tr>
<td>Translation and overturning of graphics</td>
<td>S-2-05, S-2-07</td>
<td>Spatial orientation, mental rotation</td>
</tr>
<tr>
<td>Graphics formed by one straight cut</td>
<td>S-2-02, S-2-06, S-3-01, S-3-03</td>
<td>Spatial orientation, mental rotation</td>
</tr>
<tr>
<td>Geometric modeling of one straight cut</td>
<td>S-2-02, S-2-06, S-3-01, S-3-03</td>
<td>Spatial orientation, mental rotation</td>
</tr>
</tbody>
</table>

4. Experimental Results and Discussion

4.1 Learning outcome analysis of origami of one straight cut instruction

In order to recognize the learning outcome of one straight cut instruction for elementary school fifth graders, the researcher conducted t test analysis by SPSS, comparing pretest and posttest scores of origami of one straight cut test of 41 students. Results are shown in Table 2.

Table 2. Pretest and posttest t test analysis of origami of one straight cut test

<table>
<thead>
<tr>
<th></th>
<th>Average mean</th>
<th>Standard deviation</th>
<th>t</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>63.09</td>
<td>16.80</td>
<td>-2.83</td>
<td>.007**</td>
</tr>
<tr>
<td>Posttest</td>
<td>69.30</td>
<td>17.55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* p<.05; ** p<.01

According to Table 2, the pretest average score is 63.09 with a standard deviation of 16.80. After WOOS II and physical origami learning interventions, the average posttest score is 69.30 with a standard deviation of 17.55, p=.007(p<.01). The difference is statistically significant. Students exhibit progress in posttest score, demonstrating the efficacy of WOOS II and physical origami instruction in improving students’ spatial ability.

4.2 WOOS II Performance Analysis

Students performance in WOOS II operation is analyzed next. Each of the 8 missions have full marks of 100, with points given according to the number of answered questions. Results are shown in Table 3, including the average scores and standard deviations of WOOS II operation and the average time spent on each question across all missions.

Table 3. Descriptive Statistics regarding Digital Origami of One Straight Cut Learning Sheet

<table>
<thead>
<tr>
<th>Mission</th>
<th>NO. of Questions</th>
<th>Average Scores</th>
<th>SD</th>
<th>Avg. Time Spent on Each Question (Second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mission 1</td>
<td>8</td>
<td>83.23</td>
<td>14.18</td>
<td>33.26</td>
</tr>
<tr>
<td>Mission 2</td>
<td>6</td>
<td>84.31</td>
<td>14.56</td>
<td>11.94</td>
</tr>
<tr>
<td>Mission 3</td>
<td>5</td>
<td>87.80</td>
<td>13.33</td>
<td>13.42</td>
</tr>
<tr>
<td>Mission 4</td>
<td>8</td>
<td>95.73</td>
<td>6.62</td>
<td>8.91</td>
</tr>
<tr>
<td>Mission 5</td>
<td>12</td>
<td>83.79</td>
<td>13.12</td>
<td>30.77</td>
</tr>
<tr>
<td>Mission 6</td>
<td>7</td>
<td>74.29</td>
<td>13.73</td>
<td>20.34</td>
</tr>
<tr>
<td>Mission 7</td>
<td>8</td>
<td>80.55</td>
<td>22.77</td>
<td>142.19</td>
</tr>
<tr>
<td>Mission 8</td>
<td>10</td>
<td>72.63</td>
<td>21.27</td>
<td>7.48</td>
</tr>
</tbody>
</table>

For Mission 1, the average score is 83.23 (14.18) taking on average 33.26 seconds. Mission 1 aims to examine whether the students can identify line-symmetric graphs. Mission 1 took a longer time.
for participants to complete. This is mainly because they were not yet familiar with the operating environment of WOOS II. Mission 4 received the highest scores 95.73 (6.62) taking 8.91 seconds on average. The key point of Mission 4 is to “apply the same way of folding the paper and observe whether the shapes cut out in different angles are line-symmetric graphs.” Results indicate students are familiar with the key point of Mission 4. In Mission 7, the average score is 80.55 (22.77) taking on average 142.19 seconds. The main point of Mission 7 is to “learn to present different geometric shapes, such as □ and △, with the basic shapes of origami of one straight cut”. When executing Mission 7, students performed folding and cutting movements and produced shapes with one straight cut. Thus, they could quickly tell whether or not they completed the question correctly. When students detected a wrong answer, they tried again. In this mission, the students can practice the entire process of one straight cut. These efforts, thus, consumer more time. For Mission 8, the average score is 72.63 (21.27). Mission 8 requires students to watch an animation about creating geometric shapes with one straight cut. They next need to practice predicting the final cut-out shapes. Two students answered the questions at a relatively slow speed, failing to complete some of the questions in Mission 8. The other students managed to correctly predict the unfolded shapes through the origami of one straight cut animation.

4.3 Physical Origami of One Straight Cut Learning Sheet Analysis

The eight questions of the physical origami of one straight cut learning sheet are next scored. A full score is 100 points. The average score is 69.82 (24.36) with a maximum score of 100 and a minimum score of 12.5. Performance variance is high in the physical origami of one straight cut. Results led to further analysis of the special students’ learning sheets.

This study defines students with higher than average performance, in the physical origami activities of the one straight cut learning sheet, as Excellent Students. In Figure 8, four Excellent Students, A, B, C, and D, first drew the target shapes on the colored paper, and then, determined the symmetry axes. This approach is typical of the Excellent Students. Such repeated operation produces the final correct shapes with one straight cut. More complex shapes can be obtained with the help of auxiliary lines. However, how well did Excellent Students perform in simple graphs?

![Figure 8. The Works of Excellent Students A and B for Physical Origami of One Straight Cut](image)

When operating simple graphs, Excellent Students mentally maintain the target shapes. They can identify graphic symmetry axes without drawing the shapes. This is precisely the type of skill this study has set out to examine to examine. In the hexagonal cases, as shown in Figure 9, two students did not draw the hexagonal on the colored paper in advance, because they were mentally maintaining the target shapes, allowing them to directly cut out the hexagons. It was particularly worth mentioning that Student G, when analyzing this question, drew the symmetry line of the hexagon. In this way, this student could visualize more clearly the direction and position of the paper during the actual manipulation.

![Figure 9. Excellent Students’ Hexagonal Works of Physical Origami of One Straight Cut](image)

Incorrect responses on the learning sheets are categorized into four types. The first case is shown as (a) in Figure 10. About 2% of the students failed to accomplish the shapes as indicated by the questions and directly pasted the final cut-out paper on their learning sheets. The second case is shown as (b) in Figure 10. As many as 68% of the incorrect responses fell into this category. Generally, these students folded the paper in half only once and then obtained the target shapes with several cuts (rather than the single cut as required). This occurred mostly when students did not draw the shapes on the colored paper. They did not select the correct spatial orientation due to their insufficient spatial ability. However, their operation was partially correct, meaning they clearly understood the target shapes. Therefore, when encountering difficulty in creating the target shape with one straight cut, these students decided to use multiple
4.5 Learning Performances of WOOS II and Physical Origami of One Straight Cut

This study examined how well students learned during a digital simulation (WOOS II) and then how well they performed in a physical origami of one straight cut activity. Students were divided into two groups, a high-score group and a low-score group, as based on their average scores in WOOS II operation and the physical origami activities. Student performance is divided into four categories, with results described here:

(1) WOOS II-Physical Origami low-score group: This group consisted of 8 students. Most of these students failed to complete all the missions, and had multiple failures in WOOS II operation. They tended to spend more time answering the questions because of their unfamiliarity with the line-symmetric concepts. These students could not correctly answer any questions that entailed more than two times of folding on the physical origami activity. Due to their deficiency in spatial orientation and visualization skills. These students could not identify the correct target shape after the paper was folded. As shown by the 田-shaped and 田-shaped questions. In other words, they did not look for the symmetry axis of the next graph after folding the paper in half.

(2) WOOS II low-score and Physical Origami high-score group: There were 8 students in this group. They excelled in the physical origami activities, where they could first draw the shapes on colored paper. They accomplished the tasks step-by-step. They even answered the difficult questions successfully. However, they spent a longer time in operating the WOOS II virtual system with less repeated attempts and higher error rates, indicating they were not good at practicing with the digital environment. These students had to memorize or predict the post-rotation shapes mentally, unable to use the digital representations, causing a loss of interest in practicing origami of one straight cut in the digital environment. These students exhibited more skill in physical operations, but disliked the virtual operating environment. They mastered the physical operation skills and were able to concentrate more easily on the learning of the
4.6 Discussion

WOOS II operation can help students develop and reinforce line-symmetric concepts. As described by Boakes (2009) and Cakmak, Isiksal, and Koc (2014) origami-based teaching improve the effects of student learning, and enables them the understanding of various concepts, such as line symmetry and spatial ability in an intuitive manner. However, during the WOOS II process, some students required more time to become familiar with the system interface. In addition, before WOOS II operation, physical manipulatives were applied in order to simulate the steps of origami of one straight cut, and questions were raised for students to consider. This teaching process could help students develop accurate conceptualizations. As in the origami instruction of Cakmak, Isiksal, and Koc (2014), questions raised in the learning process, for student consider, can deepen knowledge acquisition and retention. When physical manipulatives are used for simulation, the aim is to help students create a link between physical practice and virtual operation, promoting learning in a visual manner. Likewise, as mentioned by Lu and Lin (2013), origami digitalization facilitates student conceptual transformation, and repeated practices in the digital environment positively influences learning effects.

In the current study, a small number of students had a vague idea about line symmetry; however, they were only familiar with left-right symmetric graphs, not conversant with tilted symmetry axes, a common issue among learners (Tso, 2003; Liu and Liu, 1994). When operating WOOS II, these students were often confused about graphic rotation and the folding process, requiring teacher intervention. In other words, these students could not establish the folding and cutting process in their minds. A practical paper demonstration by the teacher did help these learners develop the basic symmetric concepts, and thereby, link their virtual and physical spatial abilities.

When learning the physical origami of one straight cut, students in this study answered the questions according to the three steps taught by the teacher. In the first step, the target shapes are drawn on the colored paper. The second step identifies the symmetry axes of the graphs. The final step includes one single cut when all the folded lines are overlapped. During the physical origami of one straight cut instruction, most students could accomplish the tasks following the teacher’s demonstration. However, when expected to conduct the operation on their own, students often forgot the three previously-taught steps. As a result, such students failed to mentally maintain the target shapes while folding the paper. Of course, the personal spatial orientation, spatial visualization, and mental rotation ability of each student directly influences the learning effects.

During the teaching of physical origami of one straight cut, several students in particular were observed employing different solutions for the same question. For instance, both the \( \triangle \)-shaped and \( \square \)-shaped questions had two solutions. Such students, after discussion with their peers,
deduced the key, and tried to share the skills with their fellow students. Likewise, this case also occurred in the WOOS II operation, exhibiting a growing self-efficacy among the students.

5. Conclusion

Application of WOOS II can reduce the drawbacks of hand dexterity problems and paper waste. This approach provides an auxiliary tool that allows students to repeatedly practice operations. The features of the virtual manipulatives in WOOS II emphasize displaying results in real time. During the physical origami activities, the required visual observations and hands-on operations improved finger dexterity and hand-eye-brain coordination. The physical origami activities effectively established cognition. In both virtual and physical manipulatives, the folding and cutting movements are applied in an order that students better understood the relationship between line symmetry and graphs, and are better able to identify the relationship between graphs and space. This mode of learning improves student cognition, and helps geometric, line-symmetric, and spatial concept acquisition.

When learning with virtual and physical manipulatives, students with insufficient spatial ability and line-symmetric conceptualization spend a longer time practicing, but they do not reject the teaching of virtual manipulatives. Students who excel at operating the virtual environment understand the questions and problem-solving steps within the virtual environment. Skill in physical manipulative operations gives full play to learner hand-eye-brain coordination skills, and quickly improves physical operations. Students who excel both in the virtual and physical environments are able to complete properly by using their virtual and physical spatial abilities. Moreover, some students in this study employ different methods to cut out the same target shapes, suggesting that this process enhances the self-efficacy of highly-competent and motivated students, which increases confidence and future motivation.

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